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GENERALIZED EIGENSOLUTION TO PIECEWISE CONTINUOUS DISTRIBUTED-PARAMETER MODELS OF PIEZOELECTRIC ENERGY HARVESTERS USING THE TRANSFER MATRIX METHOD

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ABSTRACT

Many multi-beam energy harvesters appearing in the literature require custom analytical or finite-element models to compute their eigensolutions and piezoelectric coupling effects. This paper discusses the use of the transfer matrix method to derive analytical solutions to beam structures with point-wise discontinuities or with lumped inertias between members or at the tip. In this method, transfer matrices are developed for the beam's states (deflection, slope, shear force, and bending moment) analogously to the state transition matrix of a linear system. Euler-Bernoulli beam theory is used to derive transfer matrices for the uniform beam segments, and point transfer matrices are derived to handle discontinuities in the structure. The transfer matrix method is shown to be advantageous for analyzing complex structures because the size of the transfer matrix does not grow with increasing number of components in the structure. Furthermore, the same formulation can be used for a wide range of geometries, including arbitrary combinations of beam segments – single- or multi-layered – and lumped inertias. The eigensolution of the transfer matrix is shown to produce the natural frequencies and mode shapes for these structures. Subsequently, the electromechanical coupling effects are incorporated and the base excitation problem is considered. The electromechanical equations of motion are decoupled by mode and shown to be a generalization of existing analytical models. Parametric case studies are provided for beam structures with varying piezoelectric layer coverage.

INTRODUCTION

A major problem with resonant vibration-based energy harvesters is that their peak power occurs near the mechanical resonant frequencies of the transducer. In order to shrink the size and mass of these devices while reducing their natural frequencies, a variety of techniques have been employed. For example, changing the standard cantilevered beam geometry and manipulating the mass distribution along the beam have been investigated. Such investigations include continuously varying cross sections [1–3], varying the ratio of tip mass to beam mass [3–4], changing the number and location of piezo patches along the beam [5–6], and multi-beam structures [7–8]. A nonlinear technique called "frequency up-conversion" also shows promise to boost power at frequencies below resonance [9–11]. Despite the prevalence of widely varying designs, no single analytic method exists for predicting the electromechanical behavior of these systems.

In the energy harvesting literature, the piezoelectric transducer is commonly modeled as a lumped, single-degree-of-freedom (DOF) system. To better predict the dynamics of energy harvesters, models have been developed based on their geometry and material properties. Two common approaches to modeling and simulating these devices are lumped parameter (typically single DOF) [12–13] and distributed parameter (multi-DOF) [13–16] models. Lumped parameter models are simple and effective when vibrating near a resonant frequency and experimental data are available to estimate the model parameters. Distributed parameter models are more accurate, can predict geometric effects such as charge cancellation, and can be easily extended to include arbitrary DOFs. However, these models are much more complex, are designed for a specific geometry, and require experimental determination of some of their parameters.